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Trace anomaly induced effective action for 2D and 4D dilaton coupled scalars

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ABSTRACT

The spherically symmetric reduction of higher dimensional Einstein-scalar theory leads to lower dimensional dilatonic gravity with dilaton coupled scalar (for example, from 4D to 2D system). We calculate trace anomaly and anomaly induced effective action for 2D and 4D dilaton coupled scalars. The large-N effective action for 2D quantum dilaton-scalar gravity is also found. These 2D results maybe applied for analysis of 4D spherical collapse. The role of new, dilaton dependent terms in trace anomaly for 2D black holes and Hawking radiation is investigated in some specific models of dilatonic gravity which represent modification of CGHS model. The conformal sector for 4D dilatonic gravity is constructed. Quantum back-reaction of dilaton coupled matter is briefly discussed (it may lead to the inflationary Universe with non-trivial dilaton).

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1 Introduction

There are few motivations to study 2D dilatonic gravity models. First of all, it is often easier to study 2D models than their 4D analogs. Especially, it may happen on quantum level. For example, for renormalizable classically solvable dilaton gravity coupled with minimal scalar matter, the problem of Hawking radiation and the back-reaction of matter to 2D black hole maybe well-understood in $1/N$ expansion [1]. Modifications of CGHS model and Hawking radiation there have been investigated in refs.[2, 3, 4] and many other works (for a review, see [5]). Second, some 2D dilatonic gravities are string inspired ones. They may serve as a laboratory for better understanding of string theory itself. Third, if one starts from the 4D Einstein-scalar or 4D Einstein-Maxwell-scalar theory then using a sherically symmetric reduction anzatz [6], one obtains the action for one of the 2D dilatonic gravity models with scalars. For example, applying such anzatz (13) to 4D Einstein-Maxwell-minimal scalar theory and integrating of the angular modes, one gets

$$\begin{aligned} S &= -\frac{1}{16\pi G}\int d^2x\sqrt{-g}\mathrm{e}^{-2\phi}\left(R+2(\nabla\phi)^2+2\mathrm{e}^{2\phi}-2Q^2\mathrm{e}^{4\phi}\right) \\ &\quad +\frac{1}{2}\int d^2x\sqrt{-g}\mathrm{e}^{-2\phi}(\nabla\chi)^2 \ . \end{aligned} \tag{1}$$

Hence, 4D spherically symmetric collapse maybe understood in terms of 2D dilatonic gravity.

In difference with CGHS model and its modifications, we have the scalar field non-minimally coupled with dilaton. Then generalization of CGHS model and study of the Hawking radiation [7] in generalized model in large-N approximation (then one has to consider N scalars in above model) requires the calculation of the trace anomaly for dilaton coupled scalar.

Such trace anomaly has been recently found for above model in ref.[8] and in case of an arbitrary dilaton-scalar coupling function $f(\phi)$ in [9]. The correspondent trace anomaly induced effective action has been also calculated [8, 9]. (Actually, the trace anomaly is proportional to b_2-coefficient of Schwinger-De Witt expansion which for general dilaton coupled scalar has been found some time ago in ref.[10]).

The natural next step is to discuss the quantum gravity contributions to such action and its applications to 2D black holes and Hawking radiation. The question of 4D generalization is also of interest.

The present paper is devoted to the study of this circle of questions. In the next section we discuss general model of dilatonic gravity [11, 12]. The trace anomaly and anomaly induced effective action are calculated for N dilaton coupled two-dimensional scalars in general form and in large-N limit. (The contribution of quantum dilaton is also included). In the case when dilaton-scalar gravity is quantized the one-loop divergent effective action is used to find the large-N non-local finite action. This action almost coincides with anomaly induced large-N action as it should be.

Section 3 is devoted to the investigation of 2D black holes in modified CGHS model (we consider dilaton coupled scalars). The new, dilaton dependent terms in trace anomaly make the problem much more complicated than for minimal case. In particular, the theory is not classically solvable anymore. The new black hole solution for purely induced theory is found. For some known cases black holes, Hawking radiation and black hole entropy are briefly discussed.

In section 4 we calculate trace anomaly and induced effective action for dilaton coupled 4D scalar. The motivation to do so is similar to 2D case. Let us start from higher dimensional Einstein-scalar theory. After spherically symmetric reduction anzats one is left with lower dimensional (say 4D) dilatonic gravity with dilaton coupled scalar.

Section 5 is devoted to the formulation of the conformal sector for 4D dilatonic gravity. The classical solutions of such theory describe quantum cosmology (with back-reaction of matter). One of the solutions for purely induced theory may correspond to the inflationary Universe. In the conclusion we give summary and list of the problems for future research.

2 One-loop effective action in the large-N approximation

We will start from the dilaton gravity of most general form [11, 12] interacting with scalar matter:

$$\begin{aligned} S &= -\int d^2x\sqrt{g}\Big\{\frac{1}{2}Z(\phi)g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi \\ &\quad +C(\phi)R + V(\phi,\chi) - -\frac{1}{2}f(\phi)g^{\mu\nu}\sum_{i=1}^{N}\partial_\mu\chi_i\partial_\nu\chi_i\Big\} \end{aligned} \tag{2}$$

It includes a dilaton field ϕ, N real dilaton coupled scalars χ_i and dilaton-dependent couplings Z, C, f. The potential V is function of ϕ and χ_i.

First of all we will be interesting in the study of the one-loop divergent effective action for the above theory. We consider two different cases. Let in the theory (2) dilaton ϕ and scalars χ_i are the quantum fields, while the gravitational field is the external field. Then one can apply the background field method [13] and to calculate the one-loop effective action [10] (see Eq.(41) of ref.[10]):

$$\begin{aligned}\Gamma_{\text{div}} &= -\frac{1}{2\epsilon}\int d^2x\sqrt{g}\Big\{\left(\frac{C''}{Z}-\frac{N+1}{6}\right)R+\frac{V''}{Z}-\frac{N}{f}\frac{\partial^2 V}{\partial\chi^2}\\ &+\left(\frac{f'^2}{2fZ}-\frac{f''}{2Z}\right)(\nabla^\lambda\chi_i)(\nabla_\lambda\chi_i)\\ &+\left(\frac{Nf''}{2f}-\frac{Nf'^2}{4f^2}--\frac{Z'^2}{4Z^2}\right)(\nabla^\lambda\phi)(\nabla_\lambda\phi)+\left(\frac{Nf'}{2f}-\frac{Z'}{2Z}\right)\Delta\phi\Big\} \end{aligned} \tag{3}$$

where $\epsilon = 2\pi(n-2)$ and we use the dimensional regularization.

The remarkable fact about the system (2) with $C = V = 0$ and the gravitational field being the classical one is that the system is conformally invariant system. Then on the quantum level the conformal (or trace) anomaly T is given by

$$\Gamma_{\text{div}} = \frac{1}{n-2}\int d^2x\sqrt{g}b_2 \qquad T = b_2 \tag{4}$$

¿From here one gets (see also [9])

$$\begin{aligned} T &= \frac{1}{24\pi}\Big\{(N+1)R--3\left(\frac{f'^2}{2fZ}-\frac{f''}{2Z}\right)(\nabla^\lambda\chi_i)(\nabla_\lambda\chi_i)\\ &-3\left(\frac{Nf''}{f}-\frac{Nf'^2}{2f^2}--\frac{Z'^2}{2Z^2}\right)(\nabla^\lambda\phi)(\nabla_\lambda\phi)--3\left(\frac{Nf'}{f}-\frac{Z'}{Z}\right)\Delta\phi\Big\}\end{aligned} \tag{5}$$

while for purely scalar field (dilaton is classical) all terms with Z in (5) disappear [9]. For a special case $N = 1$, $f(\phi) = \mathrm{e}^{-2\phi}$ and no quantum dilaton

$$T = \frac{1}{24\pi}\left\{R--6(\nabla^\lambda\phi)(\nabla_\lambda\phi)+6\Delta\phi\right\} \tag{6}$$

That trace anomaly has been recently calculated in ref.[8] using zeta-regularization method. The coefficient of third term in (6) disagrees with

the result of ref.[8]. The reasons of this disagreement have been discussed in ref.[9] (we are using not zeta-regularization but dimensional regularization).

Making the conformal transformation of metric $g_{\mu\nu} \to \mathrm{e}^{2\sigma} g_{\mu\nu}$ in the trace anomaly, using relation

$$T = \frac{1}{\sqrt{g}} \frac{\delta}{\delta\sigma} W[\sigma] \tag{7}$$

one can find anomaly induced effective action $W[\sigma]$. In the covariant, non-local form it maybe presented as following [9]:

$$\begin{aligned} W &= -\frac{1}{2} \int d^2x \sqrt{g} \Big[\frac{c}{2} R \frac{1}{\Delta} R + F_1(\phi)(\nabla^\lambda \chi_i)(\nabla_\lambda \chi_i) \frac{1}{\Delta} R \\ &\quad + \left(F_2(\phi) - \frac{\partial F_3(\phi)}{\partial\phi} \right) \nabla^\lambda \phi \nabla_\lambda \phi \frac{1}{\Delta} R + R \int F_3(\phi) d\phi \Big] \end{aligned} \tag{8}$$

where

$$\begin{aligned} c &= \frac{N+1}{24\pi}\ , \quad F_1(\phi) = -\frac{1}{8\pi} \left(\frac{f'^2}{fZ} - \frac{f''}{Z} \right), \\ F_2(\phi) &= -\frac{1}{8\pi} \left(\frac{Nf''}{f} - \frac{Nf'^2}{2f^2} - -\frac{Z'^2}{2Z^2} \right)\ , \\ F_3(\phi) &= -\frac{1}{8\pi} \left(\frac{Nf'}{f} - \frac{Z'}{Z} \right)\ . \end{aligned} \tag{9}$$

Note that for $f = \mathrm{e}^{-2\phi}$, $Z = 0$ (i.e., one has to omit all Z-dependent terms in (9)) the effective action (8) has been calculated in ref.[8]. Note also that actually it is very easy to get large-N limit of the effective action (8). To do so one only has to omit Z-dependent terms in $F_2(\phi)$, $F_3(\phi)$ and second term in c. In addition if dilaton is purely classical one should put $F_1 = 0$.

Let us consider now the theory with the action (2) as quantum dilaton-matter gravity where all fields: $g_{\mu\nu}$, ϕ and χ_i are quantized ones. Using the background field method [13]: $g_{\mu\nu} \to g_{\mu\nu} + h_{\mu\nu}$, $\phi \to \phi + \varphi$ where $h_{\mu\nu}$, φ are quantum fields, and the minimal gauge [11]

$$S_{\mathrm{gf}} = -\frac{1}{2} \int C_{\mu\nu} \chi^\mu \chi^\nu \tag{10}$$

where $\chi^\mu = -\nabla_\nu \bar{h}^{\mu\nu} + \frac{C'}{C} \nabla^\mu \varphi$, $C_{\mu\nu} = -C\sqrt{g} g_{\mu\nu}$, $\bar{h}_{\mu\nu} = h_{\mu\nu} - -\frac{1}{2} g_{\mu\nu} h$, the one-loop effective action maybe found. For pure dilaton gravity it has been obtained in ref.[11] and later in refs.[10, 14].

For dilaton-matter gravity with the classical action (2) the complete result has been obtained in ref.[10] (see Eqs.(31), (32)):

$$\begin{aligned}\Gamma_{\rm div} &= -\frac{1}{2\epsilon}\int d^2x\sqrt{g}\Big\{\frac{24-N}{6}R+\frac{2}{C}V+\frac{2}{C'}V'-\frac{V_{,ii}}{f}\\ &+\left(\frac{C''}{C}-\frac{3C'^2}{C^2}--\frac{C''Z}{C'^2}+\frac{Nf''}{2f}--\frac{Nf'^2}{4f^2}\right)(\nabla^\lambda\phi)(\nabla_\lambda\phi)\\ &+\left(\frac{C'}{C}-\frac{Z}{C'}+\frac{Nf'}{2f}\right)\Delta\phi\Big\}\end{aligned} \tag{11}$$

Using Eq.(11) one can find the one-loop effective action for any specific model.

For example, let us take

$$\begin{aligned}&Z(\phi)=4\mathrm{e}^{-2\phi}\ ,\quad C(\phi)=\mathrm{e}^{-2\phi}\ ,\\ &V(\phi,\chi)=2\ ,\quad f(\phi)=\mathrm{e}^{-2\phi}\end{aligned} \tag{12}$$

in the action (2). Then the theory (2) with dilatonic couplings (12) could be obtained (for $N=1$) by using a spherically symmetric reduction anzatz [6]:

$$ds^2=g_{\mu\nu}dx^\mu dx^\nu+\mathrm{e}^{-2\phi}d\Omega^2 \tag{13}$$

from the 4D Einstein-scalar or 4D Einstein-Maxwell-scalar theory (in the last case $V=2-2Q^2\mathrm{e}^{2\phi}$). Hence, in such case the action (2) with dilatonic couplings (12) may describe the radial modes of the extremal dilatonic black holes in four dimensions [15]. In other words, 2D dilatonic black holes may also describe 4D spherically symmetric collapse.

Using general expression (11) we may write the one-loop effective action for the theory (12) (keeping N to be an arbitrary integer):

$$\begin{aligned}\Gamma_{\rm div} &= -\frac{1}{2\epsilon}\int d^2x\sqrt{g}\Big\{\frac{24-N}{6}R+4\mathrm{e}^{2\phi}\\ &\qquad +(N-12)(\nabla^\lambda\phi)(\nabla_\lambda\phi)--N\Delta\phi\Big\}\end{aligned} \tag{14}$$

The theory is one-loop renormalizable one. Note that recently the very interesting attempt to calculate the one-loop effective action (including the local and non-local finite terms) for the model (12) has been done in ref.[16]. Unfortunately, the result [16] includes a number of mistakes. In particular, the divergent part of the one-loop effective action in the same minimal gauge

(10) of ref.[11] disagrees with the expression (14) which coincides (at least, in gravitational sector) with the independent results of refs.[11, 10, 14] in all cases where such comparison maybe done. Hence, the result of ref.[16] contradicts to those of refs.[11, 10, 14] and does not have correct on-shell limit [17]. One of the reasons of this mistake is that for the calculation of the effective action in the model (12) another dilatonic gravity classically equivalent to (12) (after conformal transformation and dilaton rescaling) is used. However, it was proved in ref.[17] (with explicit example) that classically equivalent 2D dilatonic gravities (in a sense of conformal transformation) are not quantum equivalent off-shell. They lead to different divergent one-loop effective actions which coincide only on-shell.

Let us turn again to the general model (2). In the large-N limit from (11) we get

$$\begin{aligned}\Gamma_{\rm div} &= -\frac{1}{2\epsilon}\int d^2x\sqrt{g}\Big\{-\frac{N}{6}R - -\frac{N}{f}\frac{\partial^2 V}{\partial\chi\partial\chi} \\ &+\left(\frac{Nf''}{2f} - -\frac{Nf'^2}{4f^2}\right)(\nabla^\lambda\phi)(\nabla_\lambda\phi) + \frac{Nf'}{2f}\Delta\phi\Big\} \qquad (15)\end{aligned}$$

Actually, the expression (15) is given by matter, matter-graviton and matter-dilaton loops. It maybe considered as the source for the effective trace anomaly, like in (5). Integrating such trace anomaly over σ in the same way as in Eq.(7), we will get

$$\begin{aligned}W &= -\frac{N}{2\pi}\int d^2x\sqrt{-g}\Big[\frac{1}{48}R\frac{1}{\Delta}R - -\frac{1}{8}\ln fR \\ &-\frac{1}{16\pi}\frac{f'^2}{f^2}(\nabla^\lambda\phi)(\nabla_\lambda\phi)\frac{1}{\Delta}R + \frac{1}{2Nf}\sum_{i=1}^{N}\frac{\partial^2 V}{\partial\chi_i\partial\chi_i}\mathrm{e}^{\frac{1}{\Delta}R}\Big] \,. \qquad (16)\end{aligned}$$

The expression (16) gives the large-N limit of the effective action in quantum dilatonic gravity (2). Note the appearence of new non-local term related with the scalar potential V (if it presents in the theory). Notice that V breaks the conformal invariance of the scalar field. That is why it should not be included to the system (2) when only scalars are quantized. Then few more terms of the same structure as the last one in (16) may be expected in the strict calculation of the one-loop finite effective action in dilaton-scalar gravity.

The action (8) should be used to take into account the back-reaction of quantum dilaton-matter system to classical dilatonic gravity. On the same

time the action (16) should be added to the classical dilatonic matter-gravity action if one would like to take into account the back-reaction of quantum dilaton-matter gravity (in large-N limit). The non-local actions (8), (16) open the way to new generalizations of models like CGHS-model [1] where one can find new black hole solutions and (or) new terms in the Hawking radiation. In the next section, we are going to discuss some simple properties of above effective actions in connection with $2D$ black holes.

3 2D black holes and Hawking radiation

We start with the system where the dilaton gravity of special form [1] couples with the dilaton coupled scalar fields:

$$\begin{aligned} S_0 &= \frac{1}{2\pi}\int d^2x\sqrt{-g} \\ &\times \left\{ \mathrm{e}^{-2\phi}\left[R + 4g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + 4\lambda^2\right] - -\frac{1}{2}f(\phi)\sum_{i=1}^{N} g^{\mu\nu}\partial_\mu\chi_i\partial_\nu\chi_i \right\} \end{aligned} \tag{17}$$

We would like to consider the modifications of CGHS model due to back-reaction of dilaton coupled scalars. Hence, we calculate the effective action (8) for dilaton coupled scalars in large-N approximation (gravitational field is classical field):

$$\begin{aligned} W &= -\frac{1}{2}\int d^2x\sqrt{-g}\Big[\frac{N}{48\pi}R\frac{1}{\Delta}R - -\frac{1}{8\pi}\left(\frac{f'^2}{f} - f''\right)(\nabla^\lambda\chi_i)(\nabla_\lambda\chi_i)\frac{1}{\Delta}R \\ &\quad -\frac{N}{16\pi}\frac{f'^2}{f^2}\nabla^\lambda\phi\nabla_\lambda\phi\frac{1}{\Delta}R - -\frac{N}{8\pi}\ln fR\Big] \ . \end{aligned} \tag{18}$$

Hence the complete action of our theory is given by

$$S = S_0 + W \tag{19}$$

We treat this theory as a classical system. The background scalar field is considered to be zero so we omit all scalar terms in above expression.

In the conformal gauge

$$g_{\pm\mp} = -\frac{1}{2}\mathrm{e}^{2\rho} \ , \quad g_{\pm\pm} = 0 \tag{20}$$

the equations of motion are obtained by the variation over $g^{\pm\pm}$, $g^{\pm\mp}$ and ϕ

$$
\begin{aligned}
0 &= T_{\pm\pm} \\
&= \mathrm{e}^{-2\phi}\left(4\partial_\pm\rho\partial_\pm\phi - 2\left(\partial_\pm\phi\right)^2\right) \\
&\quad +\frac{N}{12}\left(\partial_\pm^2\rho - -\partial_\pm\rho\partial_\pm\rho\right) \\
&\quad +\frac{N}{8}\left\{\left(\partial_\pm\tilde{\phi}\partial_\pm\tilde{\phi}\right)\rho + \frac{1}{2}\frac{\partial_\pm}{\partial_\mp}\left(\partial_\pm\tilde{\phi}\partial_\mp\tilde{\phi}\right)\right\} \\
&\quad +\frac{N}{8}\left\{- - 2\partial_\pm\rho\partial_\pm\tilde{\phi} + \partial_\pm^2\tilde{\phi}\right\} + t^\pm(x^\pm) \qquad (21) \\
0 &= T_{\pm\mp} \\
&= \mathrm{e}^{-2\phi}\left(2\partial_+\partial_-\phi - 4\partial_+\phi\partial_-\phi - -\lambda^2\mathrm{e}^{2\rho}\right) \\
&\quad -\frac{N}{12}\partial_+\partial_-\rho - -\frac{N}{8}\partial_+\tilde{\phi}\partial_-\tilde{\phi} - -\frac{N}{4}\partial_+\partial_-\tilde{\phi} \qquad (22) \\
0 &= \mathrm{e}^{-2\phi}\left(-4\partial_+\partial_-\phi + 4\partial_+\phi\partial_-\phi + 2\partial_+\partial_-\rho + \lambda^2\mathrm{e}^{2\rho}\right) \\
&\quad -\frac{Nf'}{f}\left\{\frac{1}{16}\partial_+(\rho\partial_-\tilde{\phi}) + \frac{1}{16}\partial_-(\rho\partial_+\tilde{\phi}) - -\frac{1}{8}\partial_+\partial_-\rho\right\}\ . \qquad (23)
\end{aligned}
$$

Here

$$\tilde{\phi} = \ln f \qquad (24)$$

and $t(x^\pm)$ is a function which is determined by the boundary condition.

First we consider the large-N limit, where classical part can be ignored. Then field equations are becoming simpler

$$
\begin{aligned}
0 &= \frac{1}{N}T_{\pm\pm} \\
&= \frac{1}{12}\left(\partial_\pm^2\rho - -\partial_\pm\rho\partial_\pm\rho\right) \\
&\quad +\frac{1}{8}\left\{\left(\partial_\pm\tilde{\phi}\partial_\pm\tilde{\phi}\right)\rho + \frac{1}{2}\frac{\partial_\pm}{\partial_\mp}\left(\partial_\pm\tilde{\phi}\partial_\mp\tilde{\phi}\right)\right\} \\
&\quad +\frac{1}{8}\left\{- - 2\partial_\pm\rho\partial_\pm\tilde{\phi} + \partial_\pm^2\tilde{\phi}\right\} + t^\pm(x^\pm) \qquad (25) \\
0 &= \frac{1}{N}T_{\pm\mp} \\
&= -\frac{1}{12}\partial_+\partial_-\rho - -\frac{1}{8}\partial_+\tilde{\phi}\partial_-\tilde{\phi} - -\frac{1}{4}\partial_+\partial_-\tilde{\phi} \qquad (26)
\end{aligned}
$$

$$0 = \frac{1}{16}\partial_+(\rho\partial_-\tilde{\phi}) + \frac{1}{16}\partial_-(\rho\partial_+\tilde{\phi}) - -\frac{1}{8}\partial_+\partial_-\rho \tag{27}$$

The function $t^\pm(x^\pm)$ can be absorbed into the choice of the coordinate and we can choose

$$t^\pm(x^\pm) = 0 \ . \tag{28}$$

Combining (25) and (26), we obtain

$$- -\frac{1}{3}(\partial_\pm\rho)^2 + \frac{1}{2}\rho(\partial_\pm\tilde{\phi})^2 - -\partial_\pm\rho\partial_\pm\tilde{\phi} = 0 \tag{29}$$

i.e.,

$$\partial_\pm\tilde{\phi} = \frac{1+\sqrt{1+\frac{2}{3}\rho}}{\rho}\partial_\pm\rho \quad \text{or} \quad \frac{1-\sqrt{1+\frac{2}{3}\rho}}{\rho}\partial_\pm\rho \ . \tag{30}$$

This tells that

$$\tilde{\phi} = \int d\rho \frac{1\pm\sqrt{1+\frac{2}{3}\rho}}{\rho} \ . \tag{31}$$

Substituting (31) into (27), we obtain

$$\partial_+\partial_-\left\{\left(1+\frac{2}{3}\rho\right)^{\frac{3}{2}}\right\} = 0 \tag{32}$$

i.e.,

$$\rho = \frac{3}{2}\left\{-1 + \left(\rho^+(x^+) + \rho^-(x^-)\right)^{\frac{2}{3}}\right\} \ . \tag{33}$$

Here $\rho^\pm$ is an arbitrary function of $x^\pm = t \pm x$. We can straightforwardly confirm that the solutions (31) and (33) satisfy (26). The scalar curvature is given by

$$\begin{aligned} R &= 8\mathrm{e}^{-2\rho}\partial_+\partial_-\rho \\ &= -\frac{8}{3}\frac{\mathrm{e}^{-3\left\{--1+\left(\rho^+(x^+)+\rho^-(x^-)\right)^{\frac{2}{3}}\right\}}}{\left(\rho^+(x^+)+\rho^-(x^-)\right)^{\frac{4}{3}}}{\rho^+}'(x^+){\rho^-}'(x^-) \end{aligned} \tag{34}$$

Note that when $\rho^+(x^+) + \rho^-(x^-) = 0$, there is a curvature singularity. Especially if we choose

$$\rho^+(x^+) = \frac{r_0}{x^+} \ , \quad \rho^-(x^-) = -\frac{x^-}{r_0} \tag{35}$$

there are curvature singularities at $x^+x^- = r_0^2$ and horizon at $x^+ = 0$ or $x^- = 0$. The asymptotic flat regions are given by $x^+ \to +\infty$ ($x^- < 0$) or $x^- \to -\infty$ ($x^+ > 0$). Therefore we can regard $x^\pm$ as corresponding to the Kruskal coordinates in 4 dimensions.

In order to discuss the Hawking radiation (which is usually related with trace anomaly [18]), it is necessary to find the exact vacuum not only at the classical level but even at the quantum level. In the following, we determine the function $\tilde{\phi} = \ln f(\phi)$ in (17) so that the linear dilaton vacuum

$$\rho = \phi = -\frac{1}{2}\left(\ln x^+ + \ln x^- + \ln \lambda^2\right) \tag{36}$$

is an exact solution. Substituting (36) into (22), we find,

$$T_{\pm\pm} = \frac{N}{16}\left((\phi')^2 + \phi''\right)\frac{\lambda^2}{x^+x^-} = 0 \tag{37}$$

This tells

$$\tilde{\phi} = 2\ln(\phi + c) \tag{38}$$

Substituting (38) into (23) we find that the constant of the integration should vanish: $c = 0$, i.e.,

$$f(\phi) = \phi^2 \ . \tag{39}$$

The solution (39) when substituted to (36) satisfies Eq.(21). If we divide the energy-momentum tensor into classical and quantum parts, the Hawking radiation is given by substituting the classical solution into the quantum part (the part proportional to N). When we substitute the shock wave solution[3]

$$\rho = \begin{cases} --\frac{1}{2}\ln\left(1 + \frac{a}{\lambda}\mathrm{e}^{\lambda\sigma^-}\right) & \sigma < \sigma_0 \\ --\frac{1}{2}\ln\left(1 + \frac{a}{\lambda}\mathrm{e}^{\lambda(\sigma^- - \sigma^+ + \sigma_0^+)}\right) & \sigma^+ > \sigma_0 \end{cases} \tag{40}$$

$$\phi = \begin{cases} --\frac{\lambda}{2}\sigma^+ - -\frac{1}{2}\ln\left(\mathrm{e}^{-\lambda\sigma^-} + \frac{a}{\lambda}\right) & \sigma^+ < \sigma_0 \\ --\frac{1}{2}\ln\left(\frac{a}{\lambda}\mathrm{e}^{\lambda\sigma_0} + \mathrm{e}^{\lambda(\sigma^+ - \sigma^-)}\right) & \sigma^+ > \sigma_0 \end{cases} \tag{41}$$

we find that ϕ dependent terms in the quantum part of the energy momentum tensor vanish when $|\sigma^+| \to \infty$. This means the behavior in the asymptotic

[3] The scalar field χ_i in (17) cannot make the shock wave. Here we suppose that the dilaton gravity also couples with another, minimal scalar field which appeared in the original CGHS model [1].

region, especially the Hawking radiation, is identical with that of the CGHS model [1].

We now investigate Bousso and Hawking's choice [8]:

$$f(\phi) = \mathrm{e}^{-2\phi} \quad (\tilde{\phi} = -2\phi) \ . \tag{42}$$

Then the quantum part of the energy-momentum tensor has the following form:

$$\begin{aligned}
T^q_{\pm\pm} &= \frac{N}{12}\left(\partial_\pm^2\rho - -\partial_\pm\rho\partial_\pm\rho\right) \\
&\quad +\frac{N}{2}\left\{\left(\partial_\pm\phi\partial_\pm\phi\right)\rho + \frac{1}{2}\frac{\partial_\pm}{\partial_\mp}\left(\partial_\pm\phi\partial_\mp\phi\right)\right\} \\
&\quad -\frac{N}{4}\left\{- - 2\partial_\pm\rho\partial_\pm\phi + \partial_\pm^2\phi\right\} + t(x^\pm) \qquad (43) \\
T^q_{\pm\mp} &= -\frac{N}{12}\partial_+\partial_-\rho - -\frac{N}{2}\partial_+\phi\partial_-\phi + \frac{N}{2}\partial_+\partial_-\phi \ . \qquad (44)
\end{aligned}$$

Substituting the classical shock wave solution (40), we find when $\sigma^+ < \sigma$

$$\begin{aligned}
T^q_{+-} &= \frac{N\lambda^2}{8}\frac{1}{\left(1+\frac{a}{\lambda}\mathrm{e}^{\lambda\sigma^-}\right)} \\
T^q_{++} &= \frac{N\lambda^2}{16}\ln\left(1+\frac{a}{\lambda}\mathrm{e}^{\lambda\sigma^-}\right) + t^+(\sigma^+) \\
T^q_{--} &= -\frac{N\lambda^2}{48}\left\{1 - \frac{1}{\left(1+\frac{a}{\lambda}\mathrm{e}^{\lambda\sigma^-}\right)^2}\right\} \\
&\quad -\frac{N\lambda^2}{16}\frac{\ln\left(1+\frac{a}{\lambda}\mathrm{e}^{\lambda\sigma^-}\right)}{\left(1+\frac{a}{\lambda}\mathrm{e}^{\lambda\sigma^-}\right)^2} + \frac{N}{16}\frac{a\lambda\mathrm{e}^{\lambda\sigma^-}\sigma^+}{\left(1+\frac{a}{\lambda}\mathrm{e}^{\lambda\sigma^-}\right)^2} + t^-(\sigma^-) \ . \qquad (45)
\end{aligned}$$

This tells that there is incoming energy from the past null infinity ($\sigma^+ \to -\infty$ or $\sigma^- \to -\infty$). However, the explicit estimation is problematic. The problem is caused by the fact that the dilaton vacuum is not the exact vacuum. Especially the last term in T^q_{--}, which is linear with respect to σ^+, tells that we cannot use the dilaton vacuum as a classical approximation. The linear term also makes impossible to determine $t^-(\sigma^-)$ by the boundary condition

at $\sigma^+ \to -\infty$ although the Hawking radiation is essentially given by $t^-(\sigma^-)$ as we will see in the following.

When $\sigma^+ > \sigma_0$, we find

$$
\begin{aligned}
T^q_{+-} &= \frac{N\lambda^2}{12}\frac{1}{\left(1+\frac{a}{\lambda}\mathrm{e}^{\lambda(\sigma^- - \sigma^+ + \sigma_0)}\right)^2} - -\frac{N\lambda^2}{6}\frac{1}{\left(1+\frac{a}{\lambda}\mathrm{e}^{\lambda(\sigma^- - \sigma^+ + \sigma_0)}\right)} \\
T^q_{\pm\pm} &= -\frac{N\lambda^2}{48}\left\{1+\frac{1}{\left(1+\frac{a}{\lambda}\mathrm{e}^{\lambda(\sigma^- - \sigma^+ + \sigma_0)}\right)^2}\right\} \\
&\quad -\frac{N\lambda^2}{16}\frac{\ln\left(1+\frac{a}{\lambda}\mathrm{e}^{\lambda(\sigma^- - \sigma^+ + \sigma_0)}\right)-1}{\left(1+\frac{a}{\lambda}\mathrm{e}^{\lambda(\sigma^- - \sigma^+ + \sigma_0)}\right)^2} + t^\pm(\sigma^\pm)
\end{aligned} \tag{46}
$$

Then when $\sigma^+ \to +\infty$, the energy momentum tensor behaves as

$$
\begin{aligned}
T^q_{+-} &\to -\frac{N\lambda^2}{12} \ , \\
T^q_{\pm\pm} &\to \frac{N\lambda^2}{48} + t^\pm(\sigma^\pm)
\end{aligned} \tag{47}
$$

This expresses the Hawking radiation but we cannot determine the unknown function $t^-(\sigma^-)$.

Hence, we found that there maybe new contributions to Hawking radiation from the dilaton dependent terms in the trace anomaly. However, in order to make their accurate estimation one has to construct new solvable models of 2D black holes with an arbitrary f and (or) another choices for dilatonic couplings Z, C and V in general model of dilatonic gravity.

Finally in this section, we evaluate the contribution to the black hole entropy from W in eq.(18). The contribution from the first classical term maybe investigated by standard methods [19]. Following this procedure , the contribution from the dilaton dependent terms

$$
- -\frac{1}{2}\int d^2x\sqrt{-g}\Big[- -\frac{N}{16\pi}\frac{f'^2}{f^2}(\nabla^\lambda\phi)(\nabla_\lambda\phi)\frac{1}{\Delta}R - -\frac{N}{8\pi}\ln fR\Big] \ . \tag{48}
$$

can be evaluated as follows

$$
- -\frac{N}{16}\int d^2x\sqrt{-g}\left(\frac{f'^2}{f^2}(\nabla^\lambda\phi)(\nabla_\lambda\phi)\psi\right) - -\frac{N}{4\pi}\ln f(\phi_0) \ . \tag{49}
$$

Here ϕ_0 is the value of the classical solution for the dilaton field at the horizon:

$$\phi_0 = -\frac{1}{2}\ln\left(\frac{M}{\lambda}\right) \tag{50}$$

and ψ is defined by

$$\Delta\psi = \delta(r) \tag{51}$$

with the boundary condition where

$$\psi \to \ln r \ , \quad \text{when } r \to 0 \ . \tag{52}$$

Here we choose the coordinate system where the metric of the black hole is given by

$$ds^2 = dr^2 + \sinh^2\sqrt{\frac{M}{\lambda}}dt^2 \ . \tag{53}$$

when Wick-rotated to the Euclidean signature. Hence, at least on qualitative level we see the appearence of new terms in quantum corrections to black hole entropy.

4 Trace anomaly and induced effective action for 4D dilaton coupled scalar

It could be interesting to generalize the results of second section for 4D case. The purpose of the present section will be to calculate the non-local effective action for 4D dilaton coupled conformal scalar. Let us consider the theory with the following Lagrangian in curved spacetime (we work in Minkowski signature)

$$L = \varphi f(\phi)(\Box - \xi R)\varphi \tag{54}$$

where φ is quantum scalar field, $\Box = g^{\mu\nu}\nabla_\mu\nabla_\nu$, ϕ is an external field (dilaton), $f(\phi)$ is an arbitrary function.

It is very easy to check that for conformal transformation

$$\begin{aligned} g_{\mu\nu} &\to \mathrm{e}^{2\sigma} g_{\mu\nu} \ , \\ R &\to \mathrm{e}^{-2\sigma}\left(R - 6\Box\sigma - 6(\nabla_\mu\sigma)(\nabla^\mu\sigma)\right) \end{aligned} \tag{55}$$

the theory with Lagrangian (54) is conformally invariant for $\xi = \frac{1}{6}$.

Let us calculate the divergent part of the effective action for the theory (54):

$$
\begin{aligned}
\Gamma_{\rm div} &= -\frac{i}{2}\mathrm{Tr}\ \ln\left\{f(\phi)\left[\Box-\xi R+\frac{\Box f(\phi)}{2f(\phi)}+\frac{(\nabla^\mu f(\phi))}{f(\phi)}\nabla_\mu\right]\right\} \\
&= \frac{1}{(n-4)}\int d^4x\sqrt{-g}b_4
\end{aligned}
\tag{56}
$$

where b_4 is the b_4-coefficient of Schwinger-De Witt expansion. The methods of its calculation are well known (see, for example, section 3.6 of ref.[13]). Applying these methods, after some algebra we will get

$$
\begin{aligned}
b_4 &= \frac{1}{2}\left(\frac{1}{6}-\xi\right)^2 R^2+\frac{1}{4}\frac{(\nabla f)^2}{f^2}\left(\frac{1}{6}-\xi\right)R+\frac{1}{32}\frac{[(\nabla f)(\nabla f)]^2}{f^4} \\
&+\frac{1}{2}\left(\frac{1}{6}-\xi\right)\Box R+\frac{1}{24}\Box\left(\frac{(\nabla f)(\nabla f)}{f^2}\right) \\
&+\frac{1}{180}\left(R^2_{\mu\nu\alpha\beta}-R^2_{\mu\nu}+\Box R\right)
\end{aligned}
\tag{57}
$$

Note that $f(\phi)$-multiplier in Eq.(56) does not give the contribution to b_4.

For $\xi=\frac{1}{6}$, we get the trace anomaly:

$$
< T_\mu^\mu >= b_4 \tag{58}
$$

Hence the trace anomaly for dilaton coupled $4D$ scalar is given by

$$
\begin{aligned}
T &= \frac{1}{(4\pi)^2}\Big\{\frac{1}{32}\frac{[(\nabla f)(\nabla f)]^2}{f^4}+\frac{1}{24}\Box\left(\frac{(\nabla f)(\nabla f)}{f^2}\right) \\
&+\frac{1}{180}\left(R^2_{\mu\nu\alpha\beta}-R^2_{\mu\nu}+\Box R\right)\Big\}
\end{aligned}
\tag{59}
$$

Here, the last term is the well-known conformal anomaly (for a review, see [20]) for conformally invariant scalar. The first two terms in (59) are the dilaton contribution to conformal anomaly.

Let us write the Eq.(59) in a slightly different form:

$$
\begin{aligned}
T &= \Big\{b\left(F+\frac{2}{3}\Box R\right)+b'G+b''\Box R+ \\
&a_1\frac{[(\nabla f)(\nabla f)]^2}{f^4}+a_2\Box\left(\frac{(\nabla f)(\nabla f)}{f^2}\right)\Big\}
\end{aligned}
\tag{60}
$$

where F is the square of Weyl tensor in four dimensions, G is Gauss-Bonnet invariant. For scalar field, it follows from (59) that

$$b = \frac{1}{120(4\pi)^2}\ , \quad b' = -\frac{1}{360(4\pi)^2}\ , \quad a_1 = \frac{1}{32(4\pi)^2}\ , \quad a_2 = \frac{1}{24(4\pi)^2} \tag{61}$$

and in principle b'' is an arbitrary parameter (it maybe changed by the finite renormalization of local counterterm).

The non-local effective action induced by the conformal anomaly (without dilaton) has been calculated sometime ago [21]. Using the equation

$$T = \frac{1}{\sqrt{-g}}\frac{\delta}{\delta\sigma}W(\sigma) \tag{62}$$

and integrating it, one can restore the non-local effective action W induced by the conformal anomaly:

$$\begin{aligned} W &= b\int d^4x\sqrt{-g}F\sigma \\ &\quad +b'\int d^4x\sqrt{-g}\Big\{\sigma\left[2\Box^2 + 4R^{\mu\nu}\nabla_\mu\nabla_\nu - -\frac{4}{3}R\Box + \frac{2}{3}(\nabla^\mu R)\nabla_\mu\right]\sigma \\ &\quad +\left(G - \frac{2}{3}R\right)\sigma\Big\} \\ &\quad -\frac{1}{12}\left(b'' + \frac{2}{3}(b+b')\right)\int d^4x\sqrt{-g}\left[R - 6\Box\sigma - -6(\nabla\sigma)(\nabla\sigma)\right]^2 \\ &\quad +\int d^4x\sqrt{-g}\Big\{a_1\frac{[(\nabla f)(\nabla f)]^2}{f^4}\sigma + a_2\Box\left(\frac{(\nabla f)(\nabla f)}{f^2}\right)\sigma \\ &\qquad +a_2\frac{(\nabla f)(\nabla f)}{f^2}[(\nabla\sigma)(\nabla\sigma)]\Big\} \end{aligned} \tag{63}$$

Here the σ-independent term is dropped, last terms represent the contribution from the dilaton dependent terms in trace anomaly. Similarly one can calculate trace anomaly and induced effective action for other theories like dilaton coupled spinor or dilaton coupled Weyl gravity with Lagrangian:

$$L = F(\phi)C_{\mu\nu\alpha\beta}C^{\mu\nu\alpha\beta} \tag{64}$$

or dilaton coupled vector: $L = -\frac{1}{4}g(\phi)F_{\mu\nu}F^{\mu\nu}$. Notice that in 2D case such vector field is not conformally invariant one.

5 Conformal sector of dilaton gravity and quantum cosmology

Let us consider now the classical theory of dilatonic gravity:

$$L_{cl} = Z(\phi) g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + C(\phi) R + V(\phi) \tag{65}$$

where Z, C, V are the arbitrary dilatonic functions. For specific choice of these functions the theory (65) represents the low-energy string effective action or Brans-Dicke gravity. So it may be considered as string-motivated classical gravity.

Adding the induced action to the action (65) (where part of linear on σ-terms are dropped away), we get in the case of conformally flat fiducial metric $g_{\mu\nu} = \mathrm{e}^{2\sigma} \eta_{\mu\nu}$:

$$\begin{aligned} S &= W + S_{cl} \\ &= \int d^4x \Big\{ 2b'(\Box\sigma)^2 - - [3b'' + 2(b+b')] \left[\Box\sigma + (\partial_\mu\sigma)^2\right]^2 \\ &\quad + a_1 \frac{[(\nabla f)(\nabla f)]^2}{f^4} \sigma + a_2 \Box \left(\frac{(\nabla f)(\nabla f)}{f^2} \right) \sigma \\ &\quad + a_2 \frac{(\nabla f)(\nabla f)}{f^2} g^{\mu\nu} (\nabla_\mu \sigma)(\nabla_\nu \sigma) \\ &\quad + \mathrm{e}^{2\sigma} Z(\phi) g^{\mu\nu} \partial_\mu\phi \partial_\nu\phi + \mathrm{e}^{2\sigma} C(\phi) \left[- - 6\Box\sigma - 6(\partial_\mu\sigma)(\partial^\mu\sigma) \right] \\ &\quad + \mathrm{e}^{4\sigma} V(\phi) \Big\} \end{aligned} \tag{66}$$

The action (66) describes the conformal sector of $4D$ dilatonic gravity. It is direct generalization of the conformal sector of $4D$ gravity which was introduced and studied in refs.[22].

That is very interesting problem for future research to study the quantum structure of the theory with the action (66), its properties, the existence of fixed points, etc. In particulary, one can expect as it happened with its analog for $\phi = \mathrm{const}$ [22] that is may provide the solution of the cosmological constant problem. The gravitational dressing of matter beta-functions in such theory may lead to the interesting consequences for Standard Model and GUTs [23].

The classical solutions of the theory (66) should define the cosmology of early universe with back-reaction of the conformal dilaton coupled matter.

However, it is not easy to search for solutions of the theory (66). (Of course, one can work again in large-N expansion what justifies the neglecting of classical term in (66)). So we will start from dilaton coupled Weyl gravity as the classical gravity. Adding to the action of such theory the induced effective action we omit the linear on σ terms. (That maybe justified by adding to the theory of dilaton and gravity dependent source for σ). Working on conformally flat metric $g_{\mu\nu} = \mathrm{e}^{2\sigma}\eta_{\mu\nu}$ (where classical action (64) is equal to zero), we may find the following classical solutions:

$$\sigma = \alpha \ln H_1 \eta \ , \quad f = \beta \ln H_2 \eta \tag{67}$$

where H_1, H_2, α and β are some constants. Their explicit values are defined by the complicated algebraic system of two equations

$$\frac{\delta W}{\delta \sigma} = 0 \ , \ \ \frac{\delta W}{\delta f} = 0 \ . \tag{68}$$

Note that for the same a_1, a_2-coefficients in W (63) one can change the coefficients b, b' and b'' by adding the conformal matter minimally interacting with the dilaton. Hence the solutions (67) define the whole class of metrics. In particular, for $\alpha = -1$, we get the solution which corresponds to the inflationary universe of Starobinsky type [24], however now with non-trivial dilaton. One can investigate other types of solutions for induced effective action, for example, black hole type solutions.

6 Summary

In summary, trace anomaly and induced action for dilaton coupled scalar in 2D and 4D dimensions are found. The large-N effective action for quantum dilaton-scalar gravity is also evaluated. The appearence of new, dilaton dependent terms in the effective action is shown. Some preliminary results on the role of these terms for 2D black holes and Hawking radiation are reported. The conformal sector of 4D dilatonic gravity is constructed and quantum cosmology is discussed.

Our results bring to the attention a number of problems. Let us mention some of them:

1. The construction of classically solvable dilaton gravities with dilaton coupled scalars. Search for new black holes in such models. Calculation of new corrections to Hawking radiation and black hole entropy.

2. Trace anomaly for 4D dilaton coupled vector, spinor and graviton. Study of quantum cosmology with back reaction of such fields.

3. Study of one-loop renormalizability of the theory (64).

4. Investigation of quantum structure for conformal sector of dilatonic gravity.

5. Generalizations of C-theorem with account of dilaton dependent terms.

We hope to return to the study of some of these questions in near future.

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